Fixed Point

Overview

When designing algorithms, it is necessary to have some basic knowledge of how the computer processes basic instructions. This is especially true with embedded applications because these processors have vastly different capabilities; the algorithm designer must account for these differences to avoid allocating superfluous amounts of memory and/or consuming excessive processor time.

Simple mathematical operations (addition, multiplication, etc.) can have a dramatic impact on the overall performance of an application, and the effect of this impact is related to the type of math that is natively supported in the processor hardware. The terminology "fixed point" often refers to the decimal point being in a fixed location for a given mathematical operation; oppositely, "floating point" implies a variable decimal point location. Floating-point processors have a device called a floating point unit (FPU) that will perform floating point operations very quickly. Fixed-point processors do not have a FPU and thus don't natively support floating-point operations; instead, floating point operations are emulated in software on a fixed-point processor and are therefore computationally expensive.

Some considerations when evaluating whether to use fixed point or floating point:

- Floating-point computations are extremely computationally expensive on a fixed-point processor.
- On a floating-point processor, floating-point operations are more efficient with respect to the number of necessary steps (don't have to apply scales/offsets, round, etc.).
- Fixed-point data types of 2 bytes or less use fewer resources (flash//RAM/EEPROM/etc.) than floating-point data types (which are at least 4 bytes).
- Floating point is convenient for quick development/testing/debugging (there is no need to protect for rollover, track scaling/offsets, etc.).

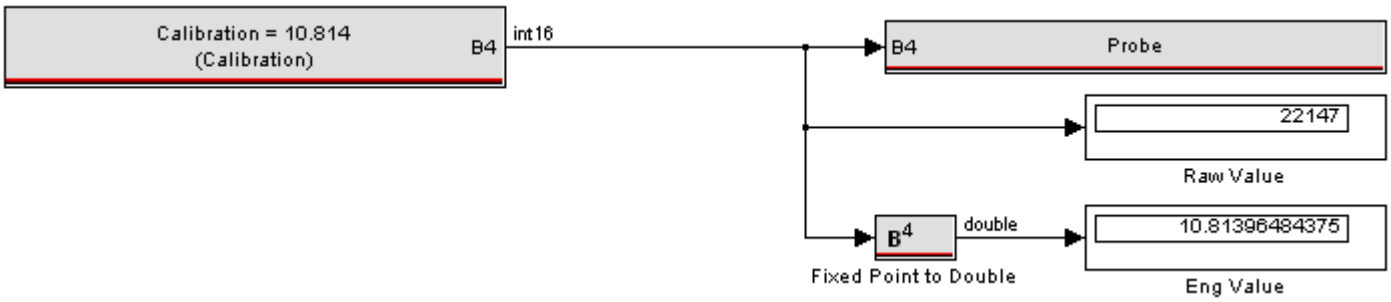
When developing for a fixed-point processor, the application is limited to integer data types (uint8, int32, etc.). However, there are techniques for managing a decimal points and resolution within a fixed-point algorithm; at some level, all of these approaches employ a gain and offset to translate the raw integer value to an engineering value that the user will observe in MotoTune. One approach uses a binary gain in conjunction with a so-called "B-Number".

B-Numbers: A Fixed-Point Approach

MotoHawk includes a block set to perform fixed-point operations using a particular property called B-Numbers. Currently, all MotoHawk Fixed Point B-Number blocks have output data types of int16. Each B-Number corresponds to a unique resolution (2^BNum / 65536) and range (spanning 65536 possible raw values) as in the table below. Note that there is no offset, so there is a trade-off on resolution for a larger range.

16-BIT SCALING				
B-Num	**Min Value**	**Max Value**	**Range**	**Resolution**
-10	-0.000976563	0.000976533	0.001953095	0.000000029802322
-9	-0.001953125	0.001953065	0.00390619	0.000000059604645
-8	-0.00390625	0.003906131	0.007812381	0.000000119209290
-7	-0.0078125	0.007812262	0.015624762	0.000000238418579
-6	-0.015625	0.015624523	0.031249523	0.000000476837158
-5	-0.03125	0.031249046	0.062499046	0.000000953674316
-4	-0.0625	0.062498093	0.124998093	0.000001907348633
-3	-0.125	0.124996185	0.249996185	0.000003814697266
-2	-0.25	0.249992371	0.499992371	0.000007629394531
-1	-0.5	0.499984741	0.999984741	0.000015258789063
0	-1	0.999969482	1.999969482	0.000030517578125
1	-2	1.999938965	3.999938965	0.00006103515625
2	-4	3.99987793	7.99987793	0.0001220703125
3	-8	7.999755859	15.99975586	0.000244140625
4	-16	15.99951172	31.99951172	0.00048828125
5	-32	31.99902344	63.99902344	0.0009765625
6	-64	63.99804688	127.9980469	0.001953125
7	-128	127.9960938	255.9960938	0.00390625

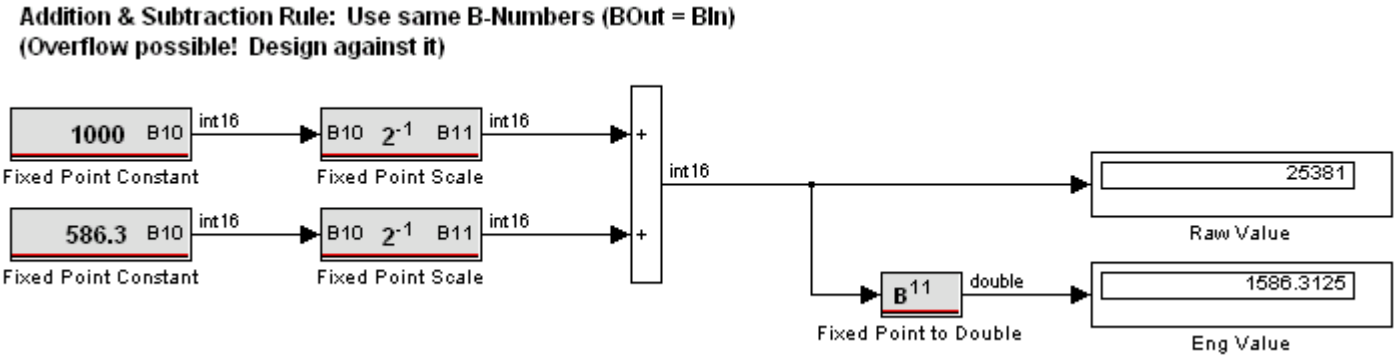
8	-256	255.9921875	511.9921875	0.0078125
9	-512	511.984375	1023.984375	0.015625
10	-1024	1023.96875	2047.96875	0.03125
11	-2048	2047.9375	4095.9375	0.0625
12	-4096	4095.875	8191.875	0.125
13	-8192	8191.75	16383.75	0.25
14	-16384	16383.5	32767.5	0.5
15	-32768	32767	65535	1
16	-65536	65534	131070	2
17	-131072	131068	262140	4
18	-262144	262136	524280	8
19	-524288	524272	1048560	16
20	-1048576	1048544	2097120	32
21	-2097152	2097088	4194240	64
22	-4194304	4194176	8388480	128
23	-8388608	8388352	16776960	256
24	-16777216	16776704	33553920	512
25	-33554432	33553408	67107840	1024



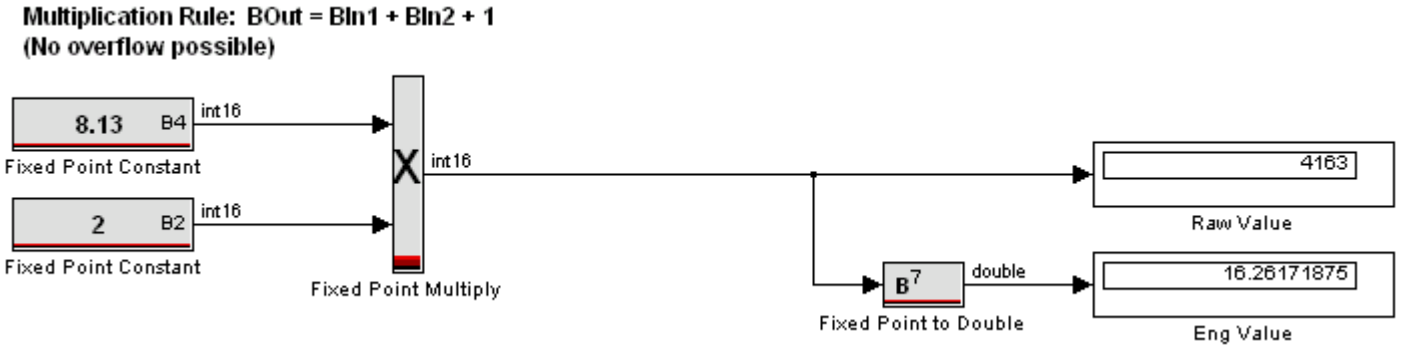
As mentioned previously, each range has 16-bit resolution that is equal to 2^BNum / 65536; thus, the scaling is of a binary type. One advantage of binary scaling over other absolute scalings is that the mathematical operations (as described subsequently) include multiplying/dividing by 2^N factors, which are actually left/right bit shifts and complete faster on the microprocessor than integer multiplies/divides. Another advantage of binary scaling in conjunction with the B-Number method is that certain rules are created that assists the application engineer in mathematical operations and preventing overflow.

Operation Rules:

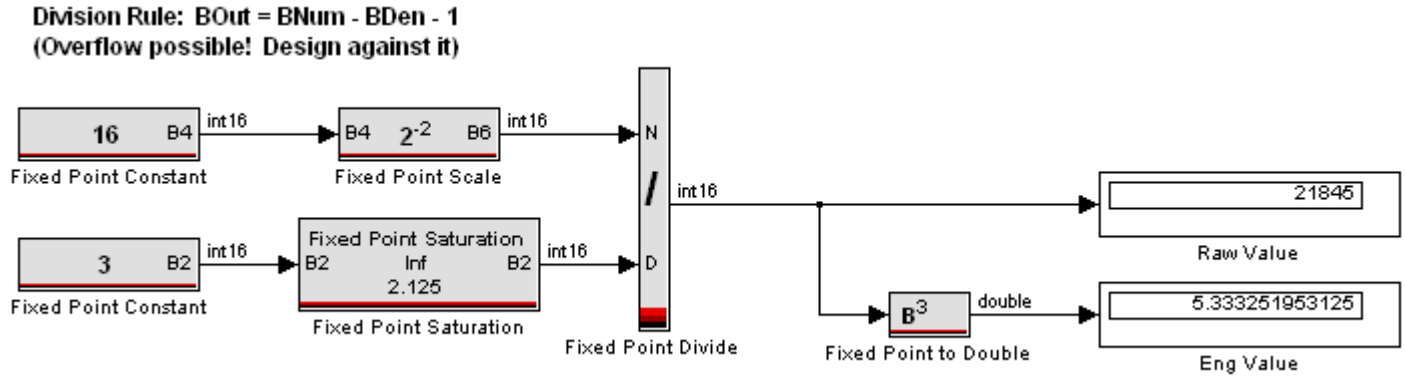
- **Addition & Subtraction**: To add/subtract, ensure the operands have the same B-Number. Note that overflow may occur; unless the design inherently prevents overflow, use a MotoHawk Fixed Point Scale block to pre-shift the operands to a higher B-Number (to increase range) prior to the operation.



- **Multiplication** (using a Motohawk Fixed Point Multiply block): The result has a B-Number equal to the sum of the B-Numbers of the operands, plus 1. Note that this rule inherently protects against overflow; no pre-shifting is necessary.



- **Division** (using a MotoHawk Fixed Point Division block): The result has a B-Number equal to the difference between the B-Numbers of the numerator and the denominator, minus 1. Note that because the denominator can approach or equal 0, overflow may occur; the design must protect against this by limiting the minimum value of the denominator and/or using a MotoHawk Fixed Point Scale block to pre-shift the numerator to a higher B-Number prior to the operation.



- **Relational Operators**: When using relational operators, ensure the operands have the same B-Number.